Bill,

Since you can answer (one hopes) all questions about the unramified principal series, let me remind you of a question I asked a long time ago.

- 1. G Chevalley group over non-archimedean local field
- 2. $G^{\widehat{0}}$ connected component of associative group
- 3. $\widehat{\mathfrak{G}}$ Lie algebra of $\overline{G^0}$.

Consider pairs $\varphi = \{\hat{t}, X\}$ up to conjugacy, $\hat{t} \in G^{\hat{0}}$, \hat{t} semi-simple. $X \in \hat{\mathfrak{G}}$, X unipotent. Ad $\hat{t}(X) = qX$, q is the number of elements in the residue field.

Can one associate to each such pair φ a finite non-empty set Π_{φ} of irreducible representations of G, which occur as constituents of the unramified principal series such that

- (i) The sets Π_{φ} are disjoint and exhaust the irreducible representations occurring in the principal series
- (ii) The sets Π_{φ} are square integrable \iff there is no proper parabolic subgroup whose Levi factor M contains t while its Lie algebra \mathcal{M} contains X. (A generalization of indecomposable representation.)
- (iii) If ξ : taking T(F), the split Cartan subgroup, to \widehat{L} is defined by

$$|\lambda(t)| = g^{(\text{There is perhaps a minus sign here.})\langle\lambda,\widehat{\lambda}\rangle} \qquad \widehat{\lambda} = \xi(t)$$

and if χ is the character of T(F) defined by

$$\chi(t) = \widehat{\lambda}(\widehat{t}) \qquad \widehat{\lambda} = \xi(t) \qquad \varphi = \{\widehat{t}, X\}$$

then each element of Π_{φ} is a constituent of $PS(\chi)$

(iv) In SL(2), there is such a pair $\hat{t} = \begin{pmatrix} q^{1/2} & 0 \\ 0 & q^{-1/2} \end{pmatrix}$, $X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. If one imbeds SL(2) in $G^{\hat{0}}$ as a principal three-dimensional subgroup Π_{φ} should consist of the special representation alone.

Bob

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